The Pioneer Anomaly: Known and Unknown Unknowns

by Donald Rums Viktor T. Toth

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The Pioneer 10/11 missions

- Launched in 1972 and 1973
- First to explore beyond Mars
- First to visit Jupiter and Saturn
- Planned duration: 600-900 days



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Mission objectives

 Primary Objectives • Explore the asteroid belt Explore beyond Mars Explore Jupiter Secondary Objectives Explore the outer solar system Search for gravity waves Search for "Planet X"



The Pioneer spacecraft



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The Pioneer spacecraft

- Mass: ~250 kg
- Radioisotope Thermoelectric Generators
- Electrical Power: ~160 W (at launch)
- 11 Scientific Instruments
- 2.75 m High Gain Antenna
- Transmitter: 8 W
- Data rate: 16-2048 bps
 Spin stabilized (4.8 rpm nominal)

Pioneer orbits – early years



Pioneer and Voyager orbits through the outer solar system



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Orientation maneuvers

- Few maneuvers needed for spinning spacecraft
- Few maneuvers → clean data
- Ingenious "Closed loop" CONSCAN maneuver lets the spacecraft "home in" on DSN signal
- Late in the mission, ~2 CONSCANs a year were performed

Pioneer 10 after 30 years

- Distance from Sun: ~80 AU
- Round-trip light time: ~21 hours
- Speed relative to the Sun: ~12 km/s

Pioneer 10 after 30 years

- One instrument (GTT) was still operating (power-down command sent last track, but never confirmed)
- Bus voltage ~ 26VDC instead of nominal 28VDC
- Transmitter XCO failed (probably due to cold)
- Transmitter still operating in coherent mode
- Many temperature readings "off scale" or outside calibrated ranges
- Propellant lines frozen (no maneuvers possible)

Pioneer 10/11 are the most precisely navigated deep space craft to date.

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The Pioneer Anomaly

- Anomalous acceleration of the Pioneer 10/11 spacecraft was detected in the 1980s, confirmed by several research teams
- May be mechanical in origin, may be "new physics"
- In the past, limited stretches of data were studied; new effort under way with complete data set, including on-board telemetry.

Discovery of the Anomaly

- Search began in 1979 (for "Planet X")
- Anomaly first detected in 1980
- Initial JPL ODP analysis in 1990-95
- Aerospace Corporation confirms: 1996-98
- Another independent confirmation by Markwardt (2002)
- Also confirmed independently by Olsen (2005), Toth (2009)

Interpreting the residual

- Frequency drift: $(5.99 \pm 0.01) \times 10^{-9}$ Hz/s (@ ~2 GHz)
- Velocity change: $(8.74 \pm 1.33) \times 10^{-10} \text{ m/s}^2$
- Clock acceleration: $(2.92 \pm 0.44) \times 10^{-18} \text{ s/s}^2$
- Velocity change (acceleration) is the "conventional" interpretation
- Effect small by engineering standards, but huge by the standards of gravity physics

Consensus as of 2006

- The Pioneer Anomaly is real
- Conventional physics fails to explain it
- Alternatives proposed include
 - Gravity modification (MOND, MSTG, Yukawa potential)
 - Dark matter
 - Cosmological origin
- $|a_P| \approx cH_0$: coincidence?

Background

- First "Pioneer Collaboration" meeting: ISSI, November 2005
- Presentation of newly recovered telemetry: complete thermal, electrical and operational record of the Pioneer 10 and 11 spacecraft
- Discussions with Slava Turyshev: No detailed thermal model for Pioneer!

Why is it important?

- Total thermal output: 2.5 kW
- Small anisotropy: -2.5% on one side, +2.5% on the other, sufficient to explain acceleration
- Thermal models are approximations
 The anisotropy is a difference that is almost 2 orders of magnitude smaller than the estimated quantities

ACCURACY IS ESSENTIAL!

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But difficult...

- Spacecraft were built 40 years ago
- Documentation is incomplete, some saved from dumpster

BUT...

- We recovered the complete telemetry record of both craft
- Telemetry is low resolution but redundant
- Sufficient documentation exists to reconstruct thermal power and material properties

What we are trying to do...

- It's not a question of either-or, but a question of how much
- Recoil force is conventional physics...

WE CANNOT IGNORE CONVENTIONAL PHYSICS!

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The case for thermal recoil

• Let me establish the case for the thermal recoil force:

The case for thermal recoil

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The case for thermal recoil

- Case has been made in 1998
- Case has been made by many others since...
- "Back of the envelope" models are a dime a dozen: P_{1→2} = ∬ P₁ cos χ₁ cos χ₂ / πr² dA₁dA₂
 Doing it the right way is hard.

The ideas are not new...

- They have been around for some time:
 - Murphy (1999): Electrical heat accounts for much of the acceleration
 - Katz (1999): Electrical heat and reflected RTG heat account for the acceleration
 - Scheffer (2003): Combination of conventional forces (including paint degradation) explain acceleration
- Dismissed using "back-of-the-envelope" estimates

Heat sources

- Heat sources are easily enumerated:
 - RTG waste heat (~2.5 kW)
 - Electrical heat (~100 W)
 - RHUs (~10 W)
 - Propulsion system (transient)

Pioneer power source



SNAP 19/PIONEER RADIOISOTOPE THERMOELECTRIC GENERATOR

RTG Thermal Power: ~650W

Electrical Power: ~40W

4 RTGs per spacecraft

~4.6 kg ²³⁸Pu on board



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Significance of spin

- Thermal forces are slowly changing. Rate of change much smaller than angular velocity: $\dot{F}/F \ll \omega/\pi$
- To first order, force components perpendicular to spin axis average to zero
- Hence only spin axis component of thermal forces needs to be computed

Linear behavior

• The two significant non-transient heat sources are electrical and RTG:

 $F \approx c^{-1} \Sigma q_i P_i$ $(P_i = P_{\text{rtg}}, P_{\text{elec}})$

- No significant trapped heat relative to the rate of change in temperatures (no latency)
- No significant variability in the emission/absorption spectrum of materials at spacecraft temperatures
- Physical configuration of spacecraft and mass constant during deep space cruise
- Temperatures are high enough
 - it can be shown that the necessary condition is $T^3 \gg k/\sigma \epsilon l$, where k is the conductance, ϵ is the emittance, l is the scale or thickness of the material, and σ is the Stefan-Boltzmann constant

Constancy and direction

- Isn't the acceleration a) constant, b) sunward?
- Short answer: No
- Long(er) answer: Acceleration is not the observable.

Long answer: ...

The Pioneer Anomaly is NOT

$a_P = (8.74 \pm 1.33) \times 10^{-10} \text{ m/s}^2$

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The Pioneer Anomaly is NOT



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instead of this:



THE PIONEER ANOMALY IS OUR INABILITY TO MODEL THE **DOPPLER RESIDUAL AT THE EXPECTED LEVEL OF ACCURACY USING ONLY KNOWN CONVENTIONAL PHYSICS.**

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The solution

 Navigators aren't doing fundamental physics. They fix the *navigational problem* by introducing fictitious forces.

The canonical solution

 A constant sunward acceleration $(a_P = (8.74 \pm 1.33) \times 10^{-10} \text{ m/s}^2)$ fixes the problem. It does NOT mean that the Pioneer spacecraft necessarily experience a constant sunward acceleration.

Other solutions

- A temporally decaying acceleration fixes the problem and it is slightly better (no statistically significant difference.)
- Earthward acceleration fixes the problem.
 - Earthward, temporally decaying acceleration fixes the problem.
- Other, equally valid solutions also exist.

The goodness of fit

- To compare solutions, we compare residuals
- Even the best residual contains plenty of noise:
 - Mismodeling of the solar system
 - Unknowns: solar plasma, troposphere, other effects
 - Unmodeled forces: small leaks
 - Measurement noise, clock stability, etc.
 - Numerical accuracy

THE PIONEER SIGNAL IS MODELED WITH AN ERROR AS LOW AS ~2 mHz OVER 20 YEARS IN A 2.29 GHz RADIO SIGNAL!

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Accuracy

 Measurement and models must be accurate to better than 1 part in 10¹⁴ over 20 years. (IEEE 64-bit double precision floating point accuracy: less than 1 part in 10¹⁶.)

Downlink power budget



Received power was $-181 \text{ dBm} (< 10^{-21} \text{ W})$ at EOM

Downlink power budget



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Noise is inevitable

- Some of it is random, some not
- Residuals have visible structure
- This explains the difference between unreasonably tight "formal errors" and realistic errors

Autocorrelation

- Statistical methods exist for estimating autocorrelation and the effective degrees of freedom (DOF) in unevenly sampled data
 - Computational difficulties
 Stability of results

 If we use a crude estimate and assume that DOF = number of model parameters, we get "realistic errors"

Effect on residuals

- Detuning the model should increase residuals
- If increase is negligible, the error bars on the detuned parameters must be correspondingly large
 What is a negligible increase?

Effect on residuals



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The "right way"

- Build a comprehensive thermal model
- Use all available data: Validate the model using redundant telemetry
- Incorporate the model into the orbit determination code to model the actual observable (Doppler)

Or the highway?

- Recovered the telemetry
- Constructed a "crude" geometric model
- Constructed a refined ray-traced model using simple isothermal surfaces but real material properties and power
- Built independent orbit determination code that incorporated the thermal recoil

A simple model



with(student); alpha=0.04; r0:=54: h:=17; d:=11; R:=evalf((r0^2+h^2)/2/h); ls[1]:=97.3; ls[2]:=102.85; [s[2]:=102.85;]s[3]:=114.95; zetas[1]:=0.097/4.*cos('delta'); zetas[2]:=0.596/4.*sin('delta'); zetas[3]:=0.596/4.*sin('delta'); for k from 1 to 3 do 1.2 f('delta'); l:=ls[k];zeta:=zetas[k]; r1:=R*Sqrt((R+d)^2+1^2)/sqrt((R+d)^2+1^2); e:=r1*sqrt((R+d)^2+1^2)/(R+d)-1*R/(R+d); r2:=r1*(R+d)/sqrt((R+d)^2+1^2); m:=sqrt((R+d)^2+1^2); s:=R-sqrt(R^2-r^2); n:=sqrt(1^2+r^2-2*1*r*cos(alpha)); u:=sqrt(n^2+(d+s)^2); coschi:=-(u^2+R^2-m^2)/(2*R*u); EQ1:= $P^{(r1^{2}(1-S)+r2^{2}S)=r1^{2}r2^{2};}$ $EQ2:=P=(r2-e)^{2}+r0^{2}-2^{*}(r2-e)^{*}r0^{*}C;$ EQ:=s/r0A2=(1-CA2)/P; use RealDomain in res1:=solve({EQ1,EQ2,EQ3,-1<=C,C<=1},{P,C,S}) end;</pre> use RealDomain in res1:=solve({EQ1,EQ2,EQ3,-1<=C,C<=1}, {P,C,S})
theta:=arccos(eval(C[],res1](1);
EQ4:=b^2*(r1^2*cos(mu)^2+r2^2*sin(mu)^2)=r1^2*r2^2;
EQ5:=b^2=(r2-e)^2q^2cos(alpha);
EQ6:=sin(mu)/q=sin(alpha)/b;
use RealDomain in res1:=solve({EQ4,EQ5,EQ6},{b,mu,q}) end:
res2:=solve(op(1,simplify(evalf(eval([q],res1)[1])),_2);
res3:= if`(evalf(eval(res2[1],alpha=Pi/2))>0,res2[1],res2[2]);
q:=unapply(factor(simplify(evalf(res3))),alpha);
heta:=arctan((ds:)n): q:=unapy+f(tation(d+s)/n); beta:=arcctan((d+s)/n); delta:=arccos((u/2+1/2-rx2-(d+s)/2)/(2*u*1)); I1:=evalf(coschi?zeta%*r/sqt(kR2-rx2)*sin(beta)/u/2/Pi): res1:=evalf(simpson(simpson(11,r=0..r0,100),a]pha=-theta,.teta,100)); res2:=evalf(simpson(simpson(11, r=0..q(alpha), 100), alpha=theta..2*Pi-theta, 100)); Pincident[k]:=Re(res1+res2); Pincioent[k]:=Re(re51+res2); I2:=evalf(coschi*zeta*R*r/sqrt(R^2-r^2)/u^2/Pi): res1:=evalf(simpson(simpson(12,r=0..r0,100),a]pha=-theta..theta,100)); res2:=evalf(simpson(simpson(12,r=0..q(a]pha),100),a]pha=theta..2*Pi-theta,100)); Preemitted[k]:=0.17*2/3*(-0.81)/0.89*Re(re51+res2); I3:=evalf(sqrt(1-r^2/R*2)*coschi*zeta*R*r/sqrt(R^2-r^2)/u^2/Pi): res1:=evalf(simpson(simpson(13,r=0..r0)0),a]pha=-theta..theta,100)); res2:=evalf(simpson(simpson(13,r=0..q(a]pha),100),a]pha=theta..2*Pi-theta,100)); red2:=evalf(simpson(simpson(13,r=0..q(a]pha),100),a]pha=theta..2*Pi-theta,100)); Pdiffuse[k]:=0.83*(1-sigma)*2/3*Re(res1+res2); uz:=d+s-2*u*coschi/R*(R-s); I4:=evalf(-uz/u*coschi*zeta*R*r/sqrt(R^2-r^2)/u^2/Pi): res1:=evalf(simpson(simpson(14, r=0..r0,100), alpha=theta..theta,100)); res2:=evalf(simpson(simpson(14, r=0..q(alpha),100), alpha=theta..2*Pi-theta,100)); Pspecular[k]:=0.83*sigma*Re(res1+res2);
Ptotal[k]:=Pincident[k]+Preemitted[k]+0.83*(Pspecular[k]+Pdiffuse[k]); end do; Ptotal[1]+Ptotal[2]+Ptotal[3]; eval(Ptotal[1]+Ptotal[2]+Ptotal[3],sigma=.7428);

0.008897107518

A more complicated model



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More results

• Thrust and spin rate change

fe elec

Iteration 1: Thrust = -36.436 / 94.8192Iteration 2: Thrust = -37.2946 / 99.5209Iteration 3: Thrust = -37.3916 / 100.052Iteration 4: Thrust = -37.403 / 100.114Iteration 5: Thrust = -37.4044 / 100.122Iteration 6: Thrust = -37.4045 / 100.122Iteration 7: Thrust = -37.4045 / 100.123Iteration 8: Thrust = -37.4045 / 100.123Iteration 9: Thrust = -37.4045 / 100.123Iteration 10: Thrust = -37.4045 / 100.123Iteration 10: Thrust = -37.4045 / 100.123

fe rtg

Iteration 1: Thrust = -0.0130118 / 0.869754, Torque(0.00721829,0.494318,0.00466599)
Iteration 2: Thrust = -0.0107575 / 0.986882, Torque(0.00146408,0.335283,0.01585)
Iteration 3: Thrust = -0.0104659 / 0.999871, Torque(-0.000599535,0.311347,0.0123182)
Iteration 4: Thrust = -0.0104445 / 1.00114, Torque(-0.000857704,0.309028,0.0117846)
Iteration 5: Thrust = -0.0104432 / 1.00125, Torque(-0.000883748,0.30882,0.0117335)
Iteration 6: Thrust = -0.0104431 / 1.00126, Torque(-0.000886127,0.308802,0.0117291)
Iteration 7: Thrust = -0.010443 / 1.00126, Torque(-0.000886442,0.3088,0.0117287)
Iteration 8: Thrust = -0.010443 / 1.00126, Torque(-0.000886456,0.3088,0.0117287)
Iteration 9: Thrust = -0.010443 / 1.00126, Torque(-0.000886456,0.3088,0.0117287)
DONE.

Spin history

Pioneer 10 spin

Pioneer 11 spin



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1996

Spin history

Pioneer 11 spin detail (1985)



Orbit determination with on-board forces

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Some results

- Confirmed the presence of the Doppler anomaly
- Verified that the recoil force is indeed a linear function of RTG and electrical heat
 - Recoil force yields good Doppler fit or conversely
- Doppler data can be used to estimate the recoil force coefficients

Still not good enough

Insufficiently detailed model (isothermal surfaces, no heat conduction, no individual instruments) Untested methodology Untested software

The real McCoy

 Report on the "definitive" model is on its way.



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The biggest known unknown

- RTG coating: "three mils of zirconia [ZrO₂] in a sodium silicate binder"
- Some similar paints gained emittance in thermal vacuum chamber tests
- Other paints lost emittance
- This specific paint was never tested
 - RTG exterior temperatures may also play a role
- A 5% decrease in emissivity can result in a 50% increase in the RTG anisotropy; a roughly 25% error in the overall thermal recoil force

So what if it is not all thermal?

• Numerical coincidences (e.g., $|a_P| \approx cH_0$) are certainly destroyed even if the sign can be explained somehow

Do not assume constancy
Do not assume direction

The sign of a_P vs. cH_0



- Much has been said about *a_P* having the wrong sign for a cosmological origin
 - This argument is not universally valid: an example is a conformal metric
 - The light of a distant star (blue) appears redshifted in accordance with Hubble's law
 - A radio signal of unit duration (half unit, actually, for drawing convenience) sent to a receding spacecraft S/C will be returned with a redshift. However, in the conformally transformed coordinate system, less time will appear to have elapsed, resulting in an apparent, small, additional blue shift. Ref: Hill, Phys. Rev. D (68) 232 (1945).

Temporal behavior

These models yield equally good residuals



Unknown direction



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Beware of traps

- All observations are two-way or three-way Doppler
- Doppler analysis is about counting beets beasts beats

Doppler measurements

One-way Doppler



Two-way Doppler



Three-way Doppler



Two-way (or three-way) Doppler



Doppler measurements



- A measurement at the receiver is made between t_1 and t_2
- These two instances of time are projected back onto the spacecraft's and then the transmitter's modeled world line; model accounts for
 - Post-Newtonian gravity of major solar system bodies
 - Maneuvers
 - Small non-gravitational forces (e.g., propellant leaks)
 - Shapiro delay
 - Effects of interplanetary medium (solar plasma)
 - Effects of the atmosphere
 - Motion of ground stations (tides, continental drift)
- The number of cycles transmitted is computed from the transmitter's known frequency
- This is then compared to the actual cycle count observed at the receiver
- Model is iteratively refined to reduce the residual difference.

Effects on the signal

- Solutions that depend on the path length (or travel time) of the signal are in trouble:
 - The geocentric velocity of the spacecraft is -20...+40 km/s The spacecraft-Earth distance is not monotonously increasing
Distance and geocentric velocity Pioneer 10



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Distance and geocentric velocity Pioneer 11



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The solar system

- Not all orbits are well known but the orbit of Saturn is known quite well thanks to Cassini
- Solution must not predict the wrong planetary orbits

Onset

- The onset is almost certainly a model artifact
- Solar mismodeling can lead to apparent onset

Onset



• At 6 AU, spacecraft still receives >225 W of solar heating

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The flyby anomaly

- Instantaneous small velocity change near perigee
- Completely unlike the Pioneer anomaly

 Quite possibly a modeling artifact: use of different, accelerating reference frames

Other spacecraft

- New Horizons: no funding for Doppler tracking; opportunity to confirm "onset" lost
- Voyagers: 3-axis stabilized
 Other spacecraft: wrong orbit, large RTGs, frequent maneuvers, etc.

Summary

- For the foreseeable future, Pioneer 10 and 11 remain the largest scale precision gravitational experiment ever conducted
- Ability to test post-Einsteinian gravity in the solar system would be marvelous
 - Far more likely, this was just a wild goose chase
 - Lessons to be learned:
 - Limits on navigational accuracy
 - Importance of preserving raw data and original documents
 - Dangers of "back of the envelope" estimation of small forces

Thank you!

• Questions?

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