COSMOLOGICAL CONSEQUENCES OF MODIFIED GRAVITY (MOG)

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International Conference on Two Cosmological Models
Universidad Iberoamericana
Ciudad de México, November 19, 2010

The need for Modified Gravity

- The ΛCDM "concordance model" works well, but...
  - It requires 96% of the universe to consist of black "stuff" that we may never be able to detect except gravitationally
  - Dark matter has difficulty even closer to home, e.g., explaining why rotational velocity follows light in spiral galaxies

- MOG fares well on many scales...
  - In the solar system or the laboratory, MOG predicts Newtonian (or Einsteinian) physics
  - The MOG acceleration law is consistent with star clusters, galaxies, and galaxy clusters

- If MOG is also consistent with cosmological data, it may be a more economical theory than ΛCDM

MOG as a field theory

- MOG is a theory of five fields:
  - The tensor field $g_{\mu\nu}$ of metric gravity
  - A scalar field G representing a variable gravitational constant
  - A massive vector field ϕ_μ (NOT a unit timelike field!) responsible for a repulsive force
  - Another scalar field μ representing the variable mass of the vector field
  - Yet another scalar field ω representing the variable coupling strength of the vector field (included for generality, but ω turns out to be constant after all)

The MOG Lagrangian

$$L = -\frac{1}{16\pi G}(R + 2\Lambda)\sqrt{-g}$$

$$-\frac{1}{4\pi}\omega\left[\frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{2}\mu^2\phi_\mu\phi^\mu + V_\phi(\phi)\right]\sqrt{-g}$$

$$-\frac{1}{G}\left[\frac{1}{2}g^{\mu\nu}\left(\frac{\nabla_\mu G\nabla_\nu G}{G^2} + \frac{\nabla_\mu\mu\nabla_\nu\mu}{\mu^2} - \nabla_\mu\omega\nabla_\nu\omega\right) + \frac{V_G(G)}{G^2} + \frac{V_\mu(\mu)}{\mu^2} + V_\omega(\omega)\right]\sqrt{-g}$$

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References:

J. W. Moffat, JCAP03(2006)004 (http://arxiv.org/abs/gr-qc/0506021)
J. R. Brownstein and J. W. Moffat, ApJ636(2006)721-741 (http://arxiv.org/abs/astro-ph/0506370)
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MOG and matter

- The MOG vector field must couple to matter
- The scalar field G must also couple to matter in specific ways to ensure agreement with solar system tests (Moffat and Toth, http://arxiv.org/abs/1001.1564)
- We specify this coupling in the case of a massive test particle by explicitly incorporating it into the test particle Lagrangian:

$$L = -m + \alpha\omega q_5 \phi_\mu u^\mu$$

MOG phenomenology

- The metric tensor is responsible for Einstein-like gravity, but G is generally greater than Newton's constant, G_N
- The vector field is responsible for a repulsive force, canceling out part of the gravitational force; the effective gravitational constant at short range is G_N
- The vector field is massive and has limited range; beyond its range, gravity is stronger than Newton predicts
- The strength of G and the range μ^{-1} of the vector field are determined by the source mass

The MOG acceleration law

- In the weak field, low velocity limit, the acceleration due to a spherically symmetric source of mass M is

$$\ddot{r} = -\frac{G_N M}{r^2}\left[1 + \alpha - \alpha(1 + \mu r)e^{-\mu r}\right]$$

- The values of α and μ are determined by the source mass M with formulas fitted using galaxy rotation and cosmology data:

$$\alpha = \frac{M}{\left(\sqrt{M} + E\right)^2}\left(\frac{G_\infty}{G_N} - 1\right), \qquad \mu = \frac{D}{\sqrt{M}},$$

$$D \cong 6250 M_\odot^{1/2}\mathrm{kpc}^{-1}, \qquad E \cong 25000 M_\odot^{1/2}, \qquad G_\infty \cong 20 G_N$$

The MOG acceleration law

- At short range, $\mu r \ll 1$, we get back Newton's acceleration law,

$$\ddot{r} = -\frac{G_N M}{r^2}$$

- At great distances, we get Newtonian gravity with an "enhanced" value of the gravitational constant:

$$\ddot{r} = -(1+\alpha)\frac{G_N M}{r^2}$$

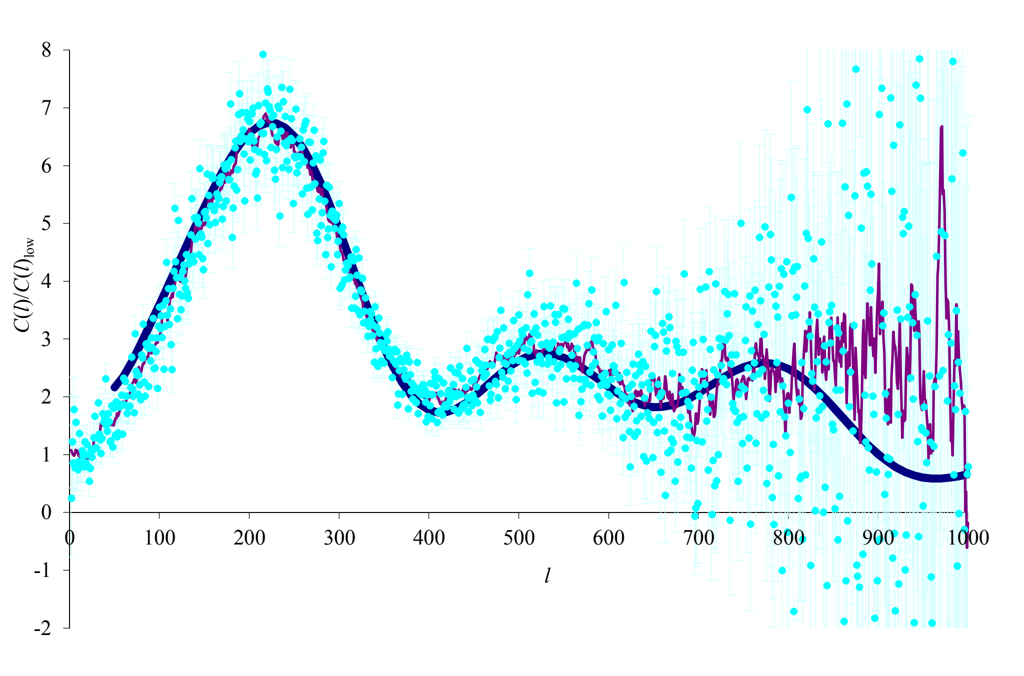
- This acceleration law is consistent with laboratory and solar system experiments, star clusters, galaxies, and galaxy clusters across (at least) 15 orders of magnitude

MOG cosmology

- We investigated the consequences of MOG in the cases of
  - The cosmic microwave background
  - The matter power spectrum
  - The luminosity-distance relationship of Type Ia supernovae

MOG and the CMB

- A key prediction (!) of ΛCDM: acoustic peaks



MOG and the CMB

- Standard questions from colleagues:
  - “Why don’t you use CMBFAST”?
  - “Why don’t you use CMB<anything>”?

MOG and the CMB

- Use of Ω in CMBFAST to represent mass densities in non-gravitational contexts makes it very difficult to use it in a variable-*G* theory:

```
          zeqp1=2.5d4*omegam*h*h*(2.7d0/tcmb)**4
    c     EH (97) fitting formula for masive neutrino growth factor.
          fnu=omegan/omegam
          fcb=(omegac+omegab)/omegam
          if (fnu.gt.0.0d0) then
             apcb=0.25d0*(5.0d0-sqrt(1.0d0+24.0d0*fcb))
             aktoq=(2.7d0/tcmb)*(2.7d0/tcmb)/(omegam*h*h)*h
             yfsok2=17.2d0*fnu*(1.0d0+0.488d0
         &        *exp(-7.0d0*log(fnu)/6.0d0))
         &        *(dble(annunr)*aktoq/fnu)**2
          else
             apcb=1.0d0
             yfsok2=0.0d0
          end if
```

MOG and the CMB

- Many “CMB<anything>”s (e.g., CMBEASY) are CMBFAST in disguise:
  - The computational engine is based on a version of CMBFAST
  - The code is often machine-translated from FORTRAN into another programming language
- If CMBFAST is not easy to modify for a variable-*G* theory, CMB<anything> is even harder

MOG and the CMB

- Mukhanov (Cambridge University Press, 2005) comes to the rescue with a semi-analytical formulation* that does not hide the physics (it is not a mere collection of fitting formulae)

- The effective gravitational constant at the horizon, $G_{\text{eff}} \cong 6G_N$, can be substituted

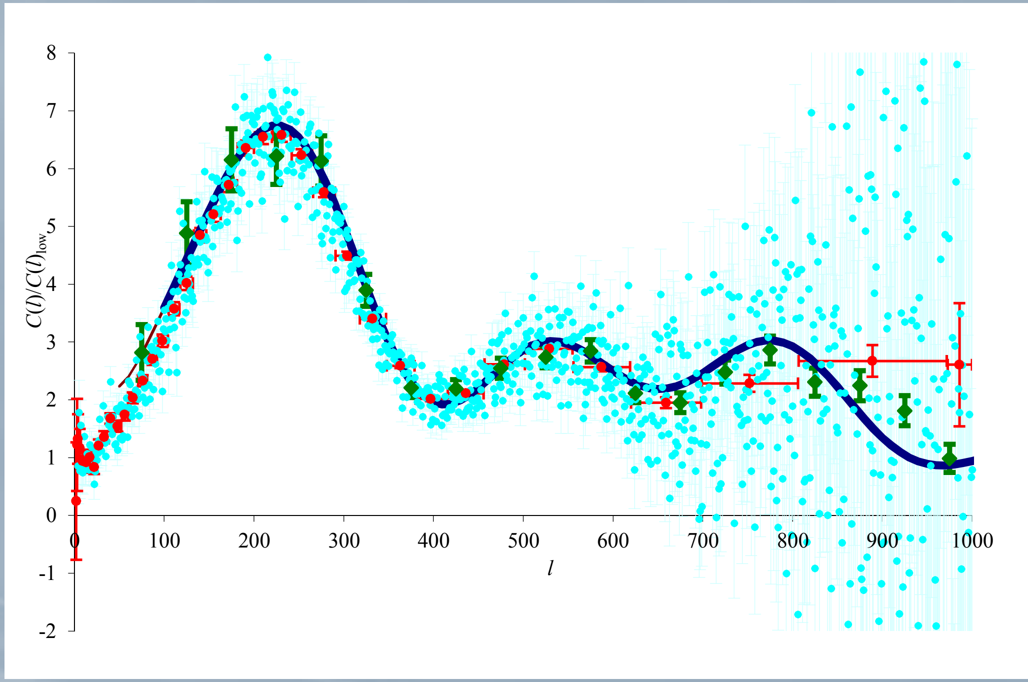
- Similarly, in the gravitational context, Ω_b can be replaced with $\Omega_M \cong 0.3$, accounting for the effects of G_{eff}

- On the other hand, when Ω_b is used in non-gravitational contexts (e.g., calculating the speed of sound), it must be left alone

*Incidentally, CMBFAST also uses semi-analytical formulations

MOG and the CMB

- The result is encouraging but not altogether surprising:
  - The enhanced gravitational constant plays the same role as dark matter in structure growth
  - Dissipation is due baryonic matter density



MOG and the matter power spectrum

- Newtonian theory of small fluctuations

$$\ddot{\delta}_{\mathbf{k}} + \frac{\dot{a}}{a}\dot{\delta}_{\mathbf{k}} + \left(\frac{c_s^2 k^2}{a^2} - 4\pi G\rho\right)\delta_{\mathbf{k}} = 0$$

for each Fourier mode $\delta = \delta_{\mathbf{k}}(t)e^{i\mathbf{k}\cdot\mathbf{q}}$ (such that $\nabla^2\delta = -k^2\delta$)

- The MOG acceleration law can be used to derive the corresponding inhomogeneous Helmholtz equation:

$$\nabla^2\Phi = 4\pi G_N\rho(\mathbf{r}) + \alpha\mu^2 G_N \int \frac{e^{-\mu|\mathbf{r}-\tilde{\mathbf{r}}|}\rho(\tilde{\mathbf{r}})}{|\mathbf{r}-\tilde{\mathbf{r}}|}d^3\tilde{\mathbf{r}}$$

MOG and the matter power spectrum

- The Helmholtz equation leads a shifting of the wave number: $k'^2 = k^2 + 4\pi(G_{\text{eff}} - G_N)\rho a^2/c_s^2$

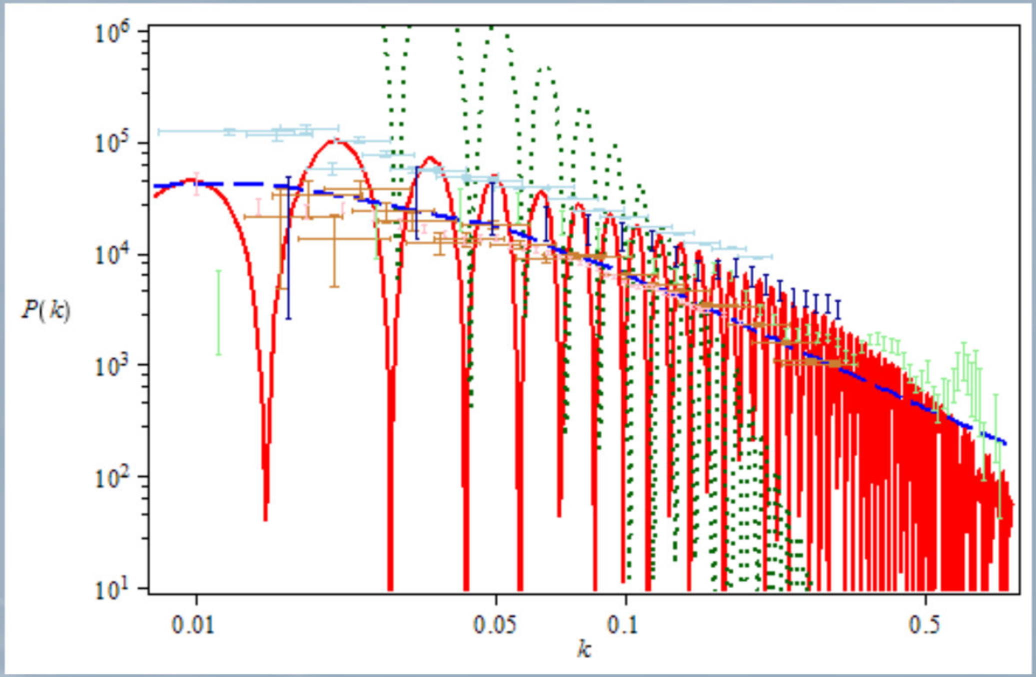
- Changes to the sound horizon scale are unaffected by the varying strength of gravity

- Silk damping introduces a $G^{3/4}$ dependence (Padmanabham, Cambridge University Press, 1993): $k'_{Silk} = k_{Silk}(G_{\text{eff}}/G_N)^{3/4}$

- These results can be used in the analytical approximations of Eisenstein and Hu (Apj496(1998)605, http://arxiv.org/abs/astro-ph/9709112)

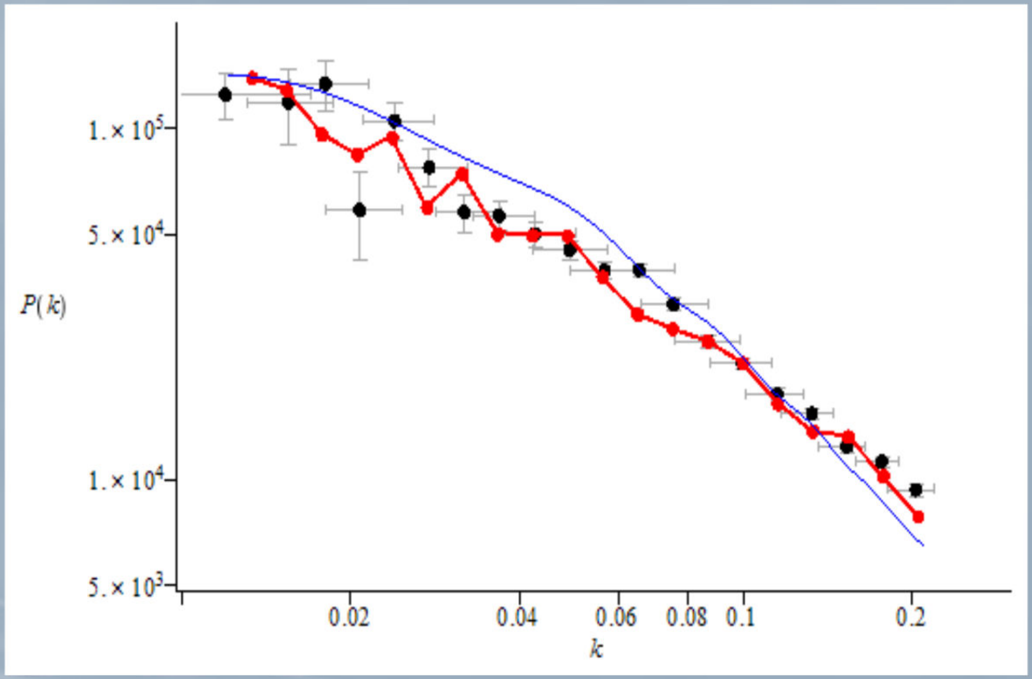
MOG and the matter power spectrum

- The result has the right slope, but in the absence of dark matter, it has unit oscillations



MOG and the matter power spectrum

- However, when we simulate the window function used in the galaxy sampling process, the oscillations are dampened



MOG and the matter power spectrum

- Two key features of the matter power spectrum are
  - Slope
  - Baryonic oscillations

- MOG produces the correct slope

- MOG has unit oscillations not dampened by dark matter

- Future galaxy surveys will unambiguously show if unit oscillations are present in the data

- The matter power spectrum can be key to distinguish modified gravity without dark matter from cold dark matter theories

MOG and continuous matter

- The CMB and matter power spectrum results were based on the MOG point particle solution
- Is it really appropriate to use the point particle solution for continuous distributions of matter? (No)
- How does MOG couple to continuous matter?
- Two constraints:
  - MOG must obey the weak equivalence principle (WEP)
  - MOG must be compatible with precision solar system observations, specifically with the values of the Eddington parameters $\beta = 1, \gamma = 1$

MOG and continuous matter

- The two Eddington parameters β and γ determine deviations from the Newtonian potential in post-Newtonian models:

$$g_{00} = 1 - \frac{2M}{r} + 2\beta\left(\frac{M}{r}\right)^2,$$
$$g_{0j} = 0,$$
$$g_{jk} = -\left(1 + \frac{2\gamma M}{r}\right)\delta_{jk}$$

- The Eddington-parameter β is identically 1 for MOG

- The Eddington-parameter γ has the same value as in JBD theory, which can be “cured” by introducing a scalar charge that makes it conformally equivalent to the minimally coupled scalar theory

MOG and continuous matter

- The WEP is often interpreted as a requirement for a metric theory of gravity, which MOG obviously isn't

- A more relaxed interpretation: the theory must be conformally equivalent to a metric theory of gravity. That is to say that there must exist a conformal transformation under which any non-minimal couplings between matter and gravity fields would vanish

- Conformal transformations add a vector degree of freedom (the special conformal transformation, a translation preceded and followed by an inversion) and a scalar degree of freedom (dilation); this agrees with the degrees of freedom to which the matter Lagrangian is expected to couple

MOG and continuous matter

- Conformal transformations:

  Dilation: $x'^{\mu} = \alpha x^{\mu}$

  Special Conformal Transformation (SCT): $x'^{\mu} = \frac{x^{\mu} - b^{\mu} x^2}{1 - 2b \cdot x + b^2 x^2}$

- The SCT can also be written as

$$\frac{x'^{\mu}}{x'^2} = \frac{x^{\mu}}{x^2} - b^{\mu}$$

- The metric is sensitive only up to a rescaling:

$$g'^{\mu\nu} = \alpha^{-2}(1 - 2b \cdot x + b^2 x^2)^2 g^{\mu\nu}$$

MOG and continuous matter

- Work-in-progress: these considerations about the WEP and γ can lead to a general prescription for the coupling between the MOG fields and matter

- We anticipate that the field equations for a perfect fluid will contain a vector charge in the form

$$\phi^\nu u_\nu J^\mu = \omega \frac{G - G_N}{G} T^{\mu\nu} u_\nu$$

and a scalar charge in the form

$$GJ = -\frac{1}{2} T$$

MOG and continuous matter

- Given an equation of state, we can write down the MOG field equations in the case of the FLRW metric,

$$ds^2 = dt^2 - a^2(t)[(1-kr^2)^{-1}dr^2 + r^2 d\Omega^2]$$

- The equations are, after setting $\omega = \text{const.}$,

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{8\pi G\rho}{3} - \frac{4\pi}{3}\left(\frac{\dot{G}^2}{G^2} + \frac{\dot{\mu}^2}{\mu^2} - \frac{1}{4\pi}G\omega\mu^2\phi_0^2\right) + \frac{2}{3}\omega G V_\phi + \frac{8\pi}{3}\left(\frac{V_G}{G^2} + \frac{V_\mu}{\mu^2}\right) + \frac{\Lambda}{3} + \frac{\dot{a}}{a}\frac{\dot{G}}{G}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{8\pi}{3}\left(\frac{\dot{G}^2}{G^2} + \frac{\dot{\mu}^2}{\mu^2} - \frac{1}{4\pi}G\omega\mu^2\phi_0^2\right) + \frac{2}{3}\omega G V_\phi + \frac{8\pi}{3}\left(\frac{V_G}{G^2} + \frac{V_\mu}{\mu^2}\right) + \frac{\Lambda}{3} + \frac{1}{2}G + \frac{1}{2}\frac{\ddot{G}}{G} - \frac{\dot{G}^2}{G^2}$$

$$\ddot{G} + 3\frac{\dot{a}}{a}\dot{G} - \frac{3}{2}\frac{\dot{G}^2}{G} + \frac{1}{2}G\frac{\dot{\mu}^2}{\mu^2} + 3\frac{V_G}{G} - V_G' + G\frac{V_\mu}{\mu^2} + \frac{1}{8\pi}G\Lambda - \frac{3}{8\pi}G\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) = -\frac{1}{2}G^2(\rho + 3p)$$

$$\ddot{\mu} + 3\frac{\dot{a}}{a}\dot{\mu} - \frac{\dot{\mu}^2}{\mu} - \frac{\dot{G}}{G}\dot{\mu} + \frac{1}{4\pi}G\omega\mu^3\phi_0^2 + 2\frac{V_\mu}{\mu} - V_\mu' = 0$$

$$\mu^2\phi_0^2 - V_\phi'(\phi)\phi_0 = 4\pi\frac{G - G_N}{G}\rho$$

MOG and continuous matter

- The FLRW field equations can be solved numerically, given suitable initial conditions and some assumptions

- We generally ignore the self-interaction potentials:

$$V_\phi = V_G = V_\mu = 0$$

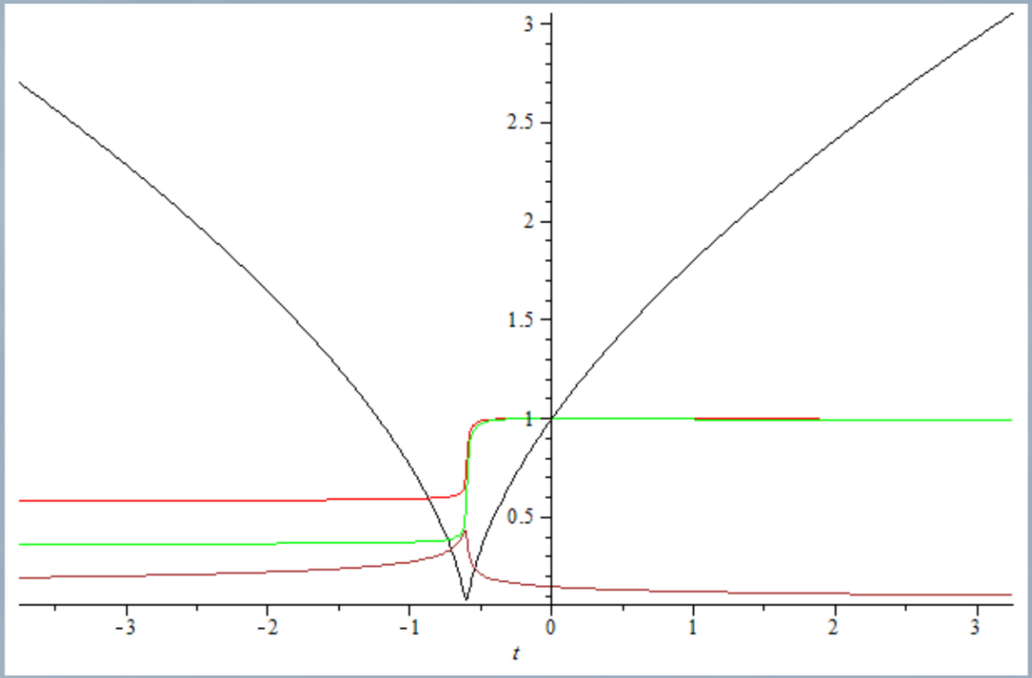
- We set the cosmological constant Λ to 0 and the curvature $k = 0$

- We assume a simple equation of state, $p = w\rho$, and we are mainly interested in the late "dust" universe, $w = 0$

- We use the present epoch to establish initial conditions: e.g., $\dot{a}/a\,|_{t=t_0} \cong 2.3 \times 10^{-18}\ \mathrm{s}^{-1}$, and $\rho|_{t=t_0} \cong 10^{-26}\ \mathrm{kg/m^3}$

MOG and continuous matter

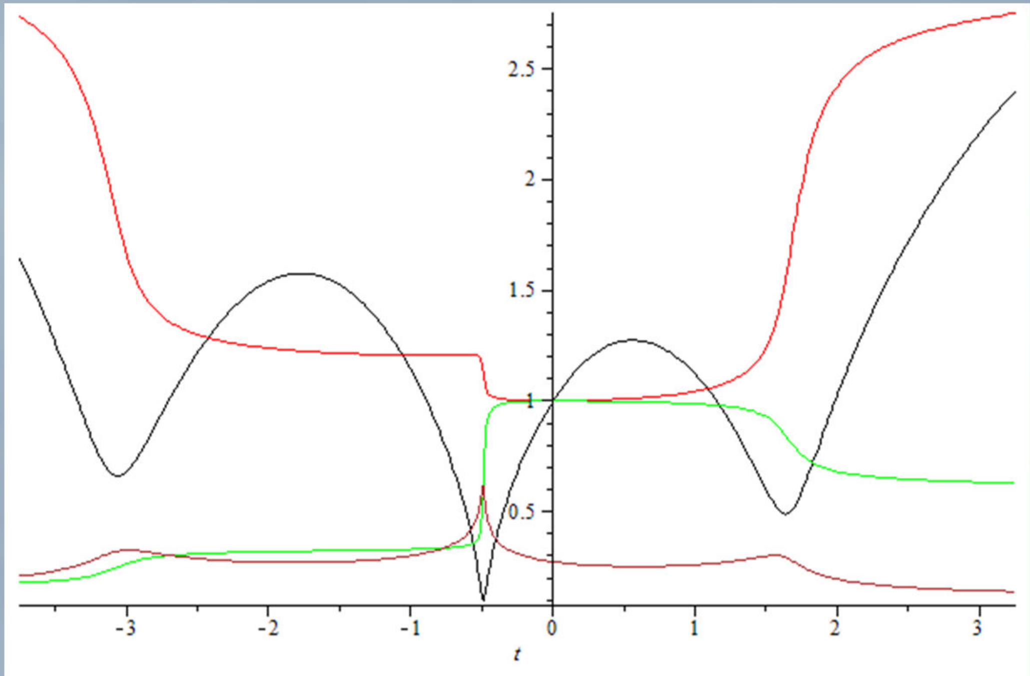
- The solution yields a “classical bounce”, albeit with an age problem:



Black is a/a_0; red is G/G_0; green is μ/μ_0; brown is $(a^3\rho)/(a_0^3\rho_0)$

MOG and continuous matter

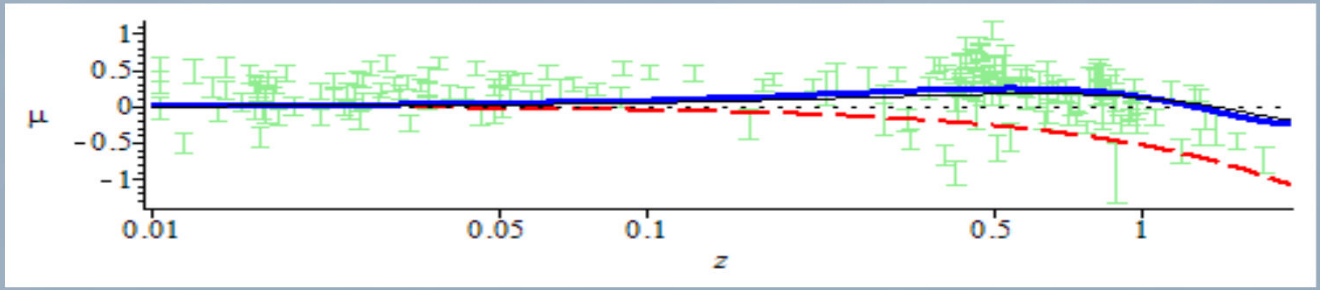
- When we set $V_G =$ const. and negative, an even more interesting picture emerges with cyclical bounces:



Black is a/a_0; red is G/G_0; green is μ/μ_0; brown is $(a^3\rho)/(a_0^3\rho_0)$

MOG and the deceleration parameter

- The deceleration parameter, $q = -\ddot{a}a/\dot{a}^2$, is 0.5 for an Einstein-de Sitter universe, but must be small (or negative) to be consistent with Type Ia supernova observations
- In the ΛCDM model, only a cosmological constant can reduce q as required
- For MOG, choosing a small positive $V_G =$ const. yields the desired result



Luminosity-redshift data of type Ia supernovae. Solid blue line is the MOG prediction. Thin black line is ΛCDM; the dashed red line is the Einstein-de Sitter universe. The horizontal dotted line corresponds to an empty universe.

Challenges

- Inflation – is it needed?

- BBN – at short range, G_{eff} is always G_N, but a variable-G theory can affect the expansion rate and abundances

- Final form of the MOG Lagrangian, with a general prescription for the coupling to matter

- Can the CMB and matter power spectrum results be reproduced without the point source solution?

- Do we really need V_G?

- N-body simulation

Conclusions

- Much work remains, but...
- MOG appears to be consistent with a large body of cosmological observations
- MOG can reproduce some precision cosmological tests
- MOG is falsifiable qualitatively (baryonic oscillations), even if detailed calculations change

Thank you!

- Any questions?